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This publication identifies some of the underlying ideas relevant to improved mathematics programs of the elementary school. This publication does not prescribe a course of study. It does, however, provide ideas to assist school personnel in their analysis of mathematics programs and textbooks. It presents an orderly outline of topics and concepts of the improved mathematics programs of the elementary school. Included are (1) some of the major mathematical content that presently constitutes elementary mathematics textbooks, and (2) a set of criteria to be considered in the selection of textbooks. (RP)

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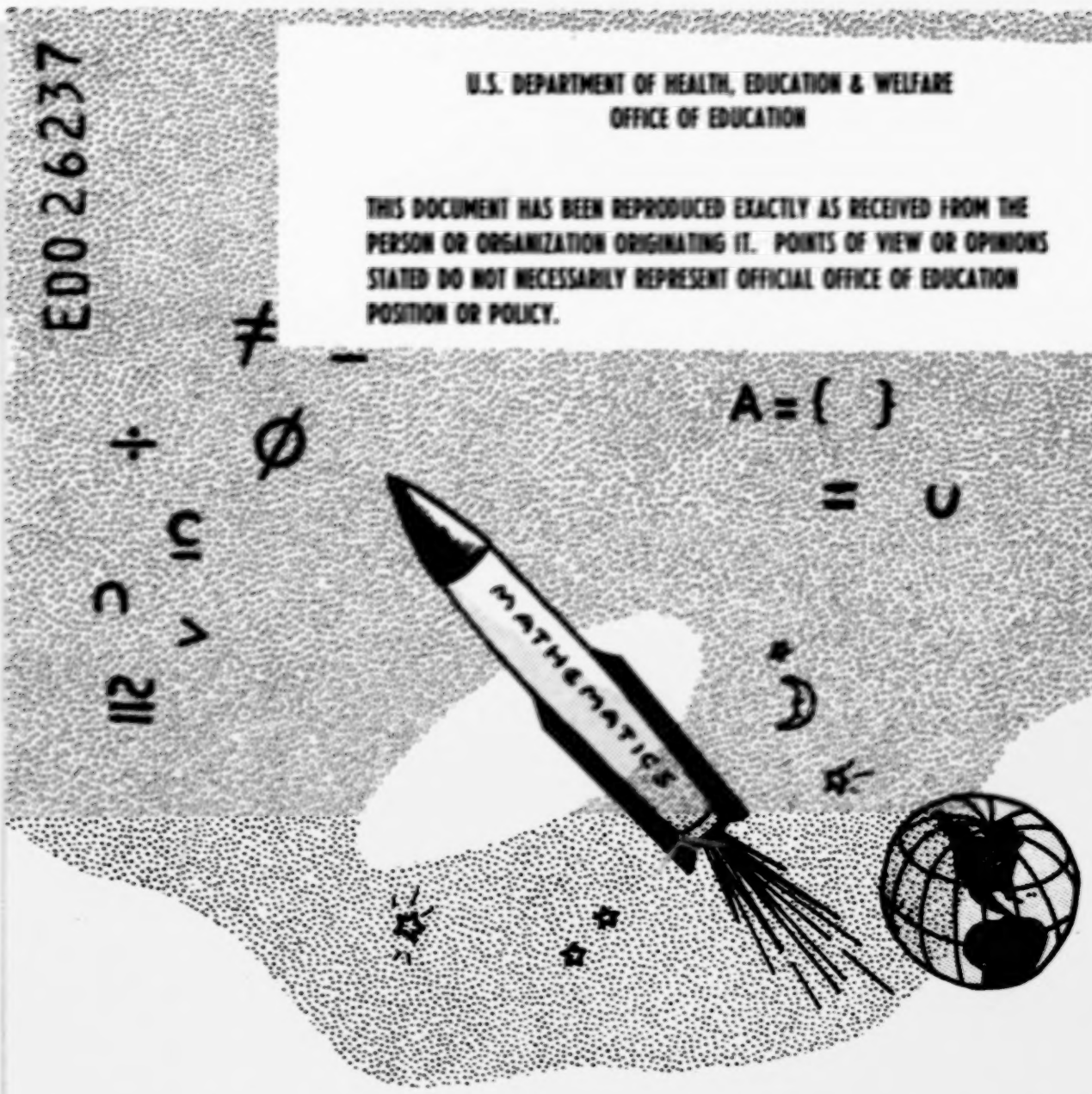

GUIDELINES
FOR
MATHEMATICS IN THE ELEMENTARY SCHOOL

SE 004 578

STATE DEPARTMENT OF EDUCATION
JESSE T. ANDERSON, STATE SUPERINTENDENT
COLUMBIA, SOUTH CAROLINA

GUIDELINES
FOR
MATHEMATICS IN THE
ELEMENTARY SCHOOL

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Jesse T. Anderson, State Superintendent
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1964

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** Reprinted with the permission of the State of California, Department of Education, George L. Roehr, Consultant in Secondary Education, Mathematics, Sacramento, California.

FOREWORD

Mathematics in our present world is continuing to make an increasing contribution to our culture as well as being an important element of scientific and cultural education. If mathematics is to continue to contribute to the advance of civilization, then it is essential that it be well selected and well taught.

Persons working in the field of education are generally aware of the development of new programs in mathematics for the elementary school. The new programs have some common objectives with previous programs but they also have additional goals and content. The adoption of new arithmetic textbooks which contain elements of the new programs to varying degrees for the public schools in South Carolina makes such programs available to every school in the state. Hence, it is highly appropriate for schools in the state to study the new programs as found in the adopted textbooks and implement that which is in harmony with the philosophy and objectives of the school. The information and suggestions in this publication will facilitate such study and evaluation.

JESSE T. ANDERSON
State Superintendent of Education

INTRODUCTION

Many schools in South Carolina are giving consideration to the improved mathematics programs now being offered by many textbooks and institutes. The recent textbook adoption by the State Board of Education gave careful attention to the needs and desires of the school districts. The textbooks adopted by the State Board of Education reflected a range of newer material and approaches to the teaching of mathematics.

As teachers move from the old to the new, a thorough study of the material that will be available and a general awareness of the content and approaches advanced by these new programs is desirable. This calls for an in-service education program and a self-study that will facilitate the transition to the newer mathematics programs.

It is the intent of this publication to identify some of the underlying ideas relevant to improved mathematics programs of the elementary school. Such ideas might well serve as a basis for in-service endeavors for the faculty group study. This publication lists some of the major mathematical content that presently constitutes elementary mathematics textbooks, and also provides a set of criteria to be considered in the selection of these textbooks. The latter part of the publication includes a resource list of professional materials. It is hoped that the content identification, criteria selection, and professional resource references will be of significant value to schools as they consider mathematics programs.

This publication does not prescribe a course of study. It does, however, provide ideas to assist school personnel in their analysis of mathematics programs and textbooks. It presents an orderly outline of topics and concepts of the improved mathematics programs for the elementary school.

J. CARLISLE HOLLER, *Director*
Division of Instruction

ACKNOWLEDGMENTS

The desire of elementary school personnel to up-date their mathematics programs and adopt textbooks in harmony with the improved programs presents a challenge. To assist schools in meeting this challenge, a publication outlining the changes and goals of the new programs and criteria for evaluating textbook materials is needed. Published materials by the State of California and the State of Washington seemed particularly relevant. With their permission this publication is possible.

Appreciation is expressed to Doctor Edwina Deans, Mathematics Specialist, United States Department of Education; Miss Dorothy Brandt, Walhalla Schools; Mrs. Sara Lindler, Brookland-Cayce Schools; Mr. Joel Taylor, Saluda Schools; Miss Frances Hudgens and Dr. Carlos Gibbons, State Department of Education, for their evaluation and suggestions pertinent to these materials.

DANIEL H. SANDEL
Supervisor for Mathematics
State Department of Education

THE CHANGING MATHEMATICS PROGRAM

New Arithmetic System Is Appealing To Children

Formula For Math: Many R Bored

Captivating Key To Math

Instant Arithmetic

Third Graders Learn Geometry

Revolutionary Arithmetic

The above are titles of newspaper and magazine articles that are being read by people across the nation. They serve only to illustrate the tremendous interest currently being shown in elementary mathematics.

Within the past four years the arithmetic program at the elementary school level has been precipitated into a state of healthy chaos. Due in part to the consequences of the swift and revolutionary changes in the mathematics program at the secondary level, but also to the first results of several research studies, a variety of improved programs has been recently developed. A few of these are complete K-6 programs; others concentrate upon new content for two or three grade levels. All, however, represent only first or initial revisions that will undergo considerable change for some time.

The goals of the elementary school program are undergoing modification. Much of the content of elementary school programs, however, remains the same. What have changed are the areas of emphasis, means of gaining understanding, some of the language of mathematics, and the placement of various topics in the curriculum pattern.

Goals for Elementary Mathematics

1. To relate mathematics instruction to the elementary curriculum in such a way that it makes an important contribution to a well-balanced total program at all grade levels.

2. To encourage the pupil to perceive the nature of number and formulate generalizations about pattern and structure.

3. To provide the student with opportunities to discover and explore functional relationships.

4. To help children appreciate mathematics as a part of their cultural heritage and to develop an understanding of mathematics as a language of description; to develop the pupil's competency and accuracy in his use of the mathematical vocabulary and the basic processes of mathematics; to develop skill in estimating an expected answer at succeeding grade levels with improved estimation resulting as the student becomes more familiar with basic properties of number and operations.

5. To relate the structure and pattern of mathematics to a wide variety of applications within the maturity level of the students so that an effective and efficient problem-solving facility will be developed.

6. To extend to each pupil an opportunity to know as much of mathematics as he has the capability and interest to learn.

Underlying Ideas in Modern Mathematics Programs

The work of study groups supported by private and public funds has given a direction to elementary mathematics programs by experimentation and development of materials. Although studies have been done independently, similar basic topics appear in the material of many of these projects. These common characteristics have become known as unifying themes, ideas, or strands, and provide bases for sequential development of mathematical knowledge, understanding, and skill. These common ideas or strands are:

1. **Concept of Sets**

   Beginning in the early grades, sets (collections or groups) are matched or their elements placed in one-to-one correspondence to develop number concepts. Later, subsets help explain problem situations and set notation extends the means for expression of mathematical ideas and relationships. Sets should be mentioned and used when appropriate.

2. **Number and Numeration**

   In the elementary school this is the area of mathematics which receives major attention and emphasis. Knowledge of numbers

and the basic operations of arithmetic are applied to develop skill in computation and problem solving. Since properties fundamental in more complex systems are basic also in numbers commonly used every day, study of these properties is a recommended part of mathematics at all elementary levels. Elementary students should recognize these ideas again and again as they study such numbers as: counting or natural numbers, whole numbers, odd numbers, even numbers, rational numbers (fractions), and integers. Activities in the early grades should teach the characteristics of our decimal system of numeration: grouping, place value, and use of zero. A study of the historical development of numeration systems lends further appreciation of the decimal system's efficient use of symbols in representing numbers and emphasizes their man-made nature. A study of other number bases in the elementary grades is useful to review and extend understanding of the decimal system.

3. Mathematical Sentences

Relations, including equality and inequality, are introduced through number sentences with emphasis on problem patterns. In solving story problems, emphasis is placed on relating the situation to patterns of mathematical operation and its statement in the form of a mathematical sentence. Symbols and understandings of other than equal relationships now taught in the early grades are: $>$ meaning is greater than; $<$ is less than; and $\neq$ is not equal to.

4. Geometry

Experience with size and space relationships begins in the kindergarten. In the middle grades words such as plane, ray, segment, congruent, etc. are introduced along with an intuitive development of mathematical ideas. In the upper grades this process continues with study of area and volume.

5. Measurement

Ideas of measure develop from experience in selecting such informal units of measure as "pencil-length," and "handspans," etc. Recognition of a need for, and knowledge of, standard units follow.

6. Proof

A young child's introduction to the nature of and need for proof can begin in his early study of numbers. These examples are given in the Twenty-Fourth Yearbook of the National Council of Teachers of Mathematics: ". . . children do prove that $7 - 3 = 4$ by showing that $4 + 3 = 7$, and . . . that $8 + 5 = 13$ because 5 is $2 + 3$, $8 + 2$ is 10 and hence $8 + 5 = 10 + 3 = 13$."[1]

On page 179 of the same reference, the chapter on proof is summarized: "In the case of probable inference, the starting point is personal experience—very convincing but unreliable—and authoritative opinion. In the case of necessary inference, it is the sentences containing 'why' and 'because' that the child makes. As he progresses through the grades, he is taught to check his judgments. He is taught to check probable inferences by securing data from experience and to check necessary inferences by applying principles of logic."[2]

7. Probability

The theory of probability is a mathematical topic applicable to uncertain or random natural events and, as with statistics, is of increasing use in modern life.

It is recommended that lessons or problems employing fundamental intuitive ideas of probability be given to students now and then as a particular topic in mathematics. Activities may include random selection of numbers (dice-casting, name-drawing, pointer-spinning) and recording the selection on charts or graphs. Enumeration of possible outcomes and development of probability ratios are experiences leading to answers to "What are the chances?" questions.

8. Statistics

This aspect of mathematics includes notions of measurement and probability with a use of symbolism. Elementary children should give attention to organization of data into tables and graphs, to understanding the median as a mid-score and the arithmetic mean as an average.

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1. National Council of Teachers of Mathematics, *The Growth of Mathematical Ideas*, Grades K-12, Twenty-Fourth Yearbook, Washington, D. C., 1959, p. 4.
2. Ibid., p. 179.

Implementation of the Program[1]

1. Examine the present program critically.
2. Study and evaluate many modern programs and materials now being suggested.
3. Plan for practical, helpful ways to provide teachers with any needed inservice training.
4. Begin to incorporate those aspects of the contemporary programs for which teachers have ample background.
5. Provide a professional library containing many books and magazines relating to mathematics.

SELECTED RESOURCE MATERIAL

Arithmetic: A Modern Approach, Youse, Bevan K. Prentice-Hall, Inc., 1963.

Arithmetic and Skills, Brumfiel, Eicholz, Shanks, O'Daffer, Addison-Wesley Publishing Co., Inc., 1963.

Arithmetic: Its Structure and Concepts, Francis Muller, Prentice-Hall, Inc., 1956.

The Arithmetic Teacher. Official Publication of the National Council of Teachers of Mathematics, Published monthly October through May.

Basic Concepts of Elementary Mathematics, William Schaaf, John Wiley and Sons, Inc., 1960.

Charting the Course for Arithmetic, Hartung, Van Engen, Knowles, Gibb, Scott-Foresman and Co., 1960.

Elementary School Mathematics, New Directions, Edwina Deans, United States Department of Health, Education and Welfare, Office of Education, Washington, D. C., 1963.

Enrichment Mathematics For The Grades. Twenty-Seventh Yearbook, National Council of Teachers of Mathematics, Washington, D. C., 1963.

Exploring Mathematics on Your Own (Series) 18 Booklets, Johnson Norton, Webster Publishing Co. 1960 and 1963.

1. See "Implementing a New Mathematics Program in Your School" by W. Eugene Ferguson, c. 4, pp. 27-33, *The Revolution in School Mathematics*, National Council of Teachers of Mathematics, Washington, D. C., 1961.

Fundamental Concepts of Elementary Mathematics, Brumfiel, Eicholz, Shanks. Addison-Wesley Publishing Co., Inc., 1962.

Geometry for Primary Grades, Books One and Two, Hawley, Suppes. Holden-Day Inc., 1961.

Glossary of Arithmetical-Mathematical Terms, Bernard H. Gundlach, Laidlaw Brothers, 1961.

Growth of Mathematical Ideas, Grades K-12, Twenty-Fourth Yearbook, National Council of Teachers of Mathematics. Washington, D. C. 1959.

Instruction in Arithmetic, Twenty-Fifth Yearbook, National Council of Teachers of Mathematics, Washington, D. C. 1960.

Introduction to Mathematics, Brumfiel, Eicholz, Shanks, Addison-Wesley Publishing Company, 1961.

Key Topics in Mathematics, Thoburn, Dye, Science Research Associates, Inc. 1962.

Learning and Teaching Arithmetic. Houston Banks, Allyn and Bacon, Inc. 1959.

Major Concepts of Elementary Modern Mathematics. Lola J. May, John Colburn Associates, Inc., 1962.

Mathematics Enrichment Series, Program A, B, C, Spooner; Program D, E. Muller, Hach, Harcourt Brace and World, 1963.

A Modern Introduction to Basic Mathematics. Mervin L. Keedy, Addison-Wesley Publishing Co., Inc. 1963.

Modern Mathematics and Your Child, Phillips, Kluttz, United States Department of Health, Education and Welfare, Office of Education, Washington, D. C. 1963.

Number in the News Series:

Arithmetic in A Rocket Age #1;
Arithmetic and Space Travel #2;
Arithmetic in A World of News #3;
Franklin Publications, Inc. 1961, 1963.

Studies in Mathematics: Volume I, *Some Basic Mathematical Concepts*, R. D. Luce. Volume V, *Concepts of Informal Geometry*, Richard D. Anderson. Volume VI, *Number Systems*. Volume VII, *Intuitive Geometry*, A. C. Vroman Inc. Pasadena, California.

The Elementary and Junior High School Mathematics Library, Clarence Hardgrove, National Council of Teachers of Mathematics, Washington, D. C., (Annotated Bibliography).

Theory of Arithmetic, Hashisaki, Peterson, John Wiley & Sons, 1963.

The Revolution in School Mathematics. National Council of Teachers of Mathematics, Washington, D. C. 1960

Elementary Arithmetic and Learning Aids, Patricia M. Spross, United States Department of Health, Education and Welfare, Office of Education, Washington, D. C., 1965

Elementary Contemporary Algebra, Ohmer, Aucoin, Cortez, Blaisdell Publishing Company, 1965.

Elementary Comtemporary Mathematics, Ohmer, Aucoin, Cortez, Blaisdell Publishing Company, 1965.

Exploration in Elementary Mathematics, Seaton E. Smith, Jr., Prentice-Hall, Inc., 1966.

Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics, Houghton Mifflin Company, 1963.

The Low Achiever in Mathematics, United States Department of Health, Education and Welfare, Office of Education, Washington, D. C., 1965.

Mathematics and the Elementary Teacher, Richard W. Copeland, W. B. Saunders Company, 1966.

Mathematics for Elementary School Teachers, National Council of Teachers of Mathematics, 1966.

Teaching Aids for Elementary Mathematics, Ethel M. Turner, Holt, Rinehart, & Winston, Inc., 1966.

Today's Mathematics, A Guide to Concepts and Methods in Elementary School Mathematics, James W. Heddens, Science Research Associates, Inc., 1964.

Topics in Mathematics for Elementary School Teachers, Twenty-ninth Yearbook, National Council of Teachers of Mathematics, 1964.

CURRENT ELEMENTARY TEXTBOOK CONTENT

The material contained in this section is a composite of the contents of elementary school arithmetic textbooks examined by the Instructional Materials Subcommittee of the Washington State Mathematics Advisory Committee. This summary was prepared by Miss Dorine Guthrie, Associate Professor of Mathematics, Eastern Washington State College.

STRANDS	KINDERGARTEN
1. Concepts of Sets	Familiar objects "Chairs to pupils" "Erasers to chalk"
2. Number and Numeral	Natural order of numbers, recognition of sets and their cardinal number of 1, 2, 3, 4, 5, 6 First, second, third, first and last
3. Enumeration systems a. Place value b. Expanded notation	Counting
4. Geometry	Recognition of shapes, □, △, ○, ▭
5. Properties & techniques of operations a. Natural numbers b. Whole numbers c. Fractions d. Integers	Union of sets such as △ △ △ + △ = ○ 3 + 1 = 4 □ □ □ □ + ○ = □ □ □ □ □ □ 4 + 2 = 6
6. Equalities & inequalities a. Sentences b. Symbols Other patterns	Visualize: larger, smaller, greater than, less than One more than patterns: 1 ○, 2 ○ ○, 3 ○ ○ ○
7. Measurement	Concept of temperature, warmer or cooler; taller, shorter; longer, shorter, time school starts, etc.
8. Statistics a. Graphs Pictographs Circles Lines Bar Coordinates	Pictographs 3

STRANDS	FIRST GRADE
1. Concepts of Sets	One-to-one correspondence Equivalent and non-equivalent, union, subsets Set separation
2. Number and Numeral	Concept of Fractional Numbers, ½, ⅓, ¼ Abstracting cardinal numbers from equivalent sets Ordinal numerals first through tenth Understanding numbers 0 - 100 Numerals as names for numbers, reading and writing numerals, 0 - 100; different numerals for a number Counting by ones, twos, fives, and tens
3. Enumeration systems	
a. Place value	Use of ten digits, 0 - 9, use of ten symbols Place value numerals 0 - 100
b. Expanded notation	Expanded notation 83 = 8 tens + 3 ones
4. Geometry	Recognition of geometric forms, point, line closed curves ○, △, □, ▭
5. Properties & techniques of operations	
a. Natural numbers b. Whole numbers c. Fractions	Addition of whole numbers, commutative and associative properties. Additive property of 0, Two digit addition and subtraction combinations through 18
d. Integers	Expanded notation, vertical notation Subtracting zero Subtracting a number from itself
6. Equalities & inequalities	
a. Sentences	Comparing numbers
b. Symbols	Relations symbols $<$, $>$, $=$, $\neq$
Other patterns	Natural order of whole numbers, "One-more" $3 + 1 = 4$ "One-less" $4 - 1 = 3$ Recognition of signs of operations and order $3 + 4 \bigcirc 4 + 2$ Finding the missing addend $3 + \square = 7$; $\square + 4 = 7$; $7 - 3 = \square$; $7 - 4 = \square$; $16 + 4 = \square$; $20 - \square = 16$ Story problems

7. Measurement	Money–penny, nickel, dime Time–hour, half hour, day, week, month Concept of unit measure Concept of linear measure, 1 inch, 1 foot Capacity measure–cup, pint, quart Reading temperatures in degrees Fahrenheit
8. Statistics	
a. Graphs Pictographs Circles Lines Bar Coordinates	Pictograph

STRANDS	SECOND GRADE
1. Concepts of Sets	Set notation $\{1, 2, 3\}$; Empty set $\{\quad\}$; Union of disjoint sets as addition
2. Number and Numeral	Fractional numbers to include $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{1}{6}$– Review of cardinal number of a set Understanding reading & writing numerals, 0-1000 Counting by threes, fours, sixes, sevens, eights, nines Ordinal number related to counting numbers
3. Enumeration systems	
a. Place value	Use of set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ Place value numerals, 0 - 1000
b. Expanded notation	Expanded notation to thousands 3251 = 3 thousands + 2 hundreds + 5 tens + 1 one
4. Geometry	Line segments; number line as model of whole number Order relation of numbers on number line
5. Properties & techniques of operations	
a. Natural numbers	

b. Whole numbers	Whole numbers – 1, 2, 3 digit numbers; Regroup sets – Do informal "carrying," expanded notation Find missing addend, recogntion of missing operational and order symbol
c. Fractions	Parts of whole, equivalent subsets
d. Integers	
6. Equalities & inequalities	
a. Sentences	Mathematical sentences
b. Symbols	Finding the sum $324 + 268 = \triangle$
Other patterns	3 hundreds + 2 tens + 4 ones 2 hundreds + 6 tens + 8 ones 5 hundreds + 8 tens + 12 ones = 5 hundreds + 9 tens + 2 ones
	Missing addend, $24 + \triangle = 30$
	Recognition of operation and order relation $422 \bigcirc 222$; $422 \bigcirc 200 + 200 + 20 + 2$
7. Measurement	Money–quarter, half dollar; time–minutes, year Clock arithmetic, addition and subtraction Linear measure to ¼ inch, yard Concept of capacity measure, ½ pint, gallon Concept of weight–pounds
8. Statistics	
a. Graphs	Continued pictographs $3 + 4$
Pictographs	Graphs on the number line $3 + 4 = 7$ .../....
Circles	
Lines	
Bar	
Coordinates	

STRANDS	THIRD GRADE
1. Concepts of Sets	Union $\cup$; Intersection $\cap$; Solution Sets
2. Number and Numeral	Fractional numbers – ⅗ as $3 \times \frac{1}{5}$, etc
3. Enumeration systems	Place value numerals 0 – 1,000,000
a. Place value	Expanded notation to millions
b. Expanded notation	

4. Geometry

Same as grade 2, extended to include fractions on the number line, fourths, thirds, halves

Ideas of separation–Point separates a line into two half lines

5. Properties & techniques of operations

a. Natural numbers
b. Whole numbers
c. Fractions
d. Integers

Inverse of multiplication $3 \times \triangle = 12$, $12 \div 3 = \triangle$

Repeated subtractions, addition and subtraction to include seven digit numbers;

Multiplication

Sets

4 sets of 3,	xxx xxx xxx xxx	= 12
3 sets of 4,	xxxx xxxx xxxx	= 12

Commutative property

Arrays

3 X 4	xxxx xxxx xxxx
4 X 3	xxx xxx xxx xxx

Development of multiples of 0-9; factors of a number

Two factors alike, $5 \times 5 = 25$; squaring

Square root, then the square root of 25 is 5

Identity element of one

Associative property

$$5(84) = 5 \times (2 \times 42) = (5 \times 2) \times 42$$
$$= 10 \times 42 = 420$$

Distributive property

$$5(39) = 5(30 + 9) = 5(30) + 5(9)$$

Division – sets, 2 sets of 4 in 8 $\quad 8 \div 4 = 2$

Arrays – $3 \times 6 = 18$, $\quad 18 \div 6 = 3$

6. Equalities & inequalities

a. Sentences

To include multiplication and division, numbers through 3 digit

b. Symbols

Addition, subtraction through millions from story problems

$3N = 27$, find N; $\quad \frac{1}{3}N = 9$, find N.

Other patterns

Recognition of number sequence patterns,
0, 1, 1, 2, 3, 5, 8, 13 · · ·; 0, 6, 12, 18, 24 · · ·

7. Measurement	Money--introduction of dollar sign & decimal point
8. Statistics	
a. Graphs	Continued pictographs
Pictographs	One dimension line graphs including fractions
Circles	
Lines	
Bar	
Coordinates	

STRANDS	FOURTH GRADE
1. Concepts of Sets	Extended work on subsets, union of disjoint sets as addition, intersection of sets of points, sets of lines
2. Number and Numeral	Fractional numerals, diagrams, sets and a number line $2\frac{1}{2} = 3 - \frac{1}{2} = 2 + \frac{1}{2} = 5 \times \frac{1}{2}$
3. Enumeration systems	
a. Place value	Base 5 numerals $\{ 0, 1, 2, 3, 4 \}$; Roman system of numeration
b. Expanded notation	Expanded notation 324 five = 3 (five X five) = 2 (five) + 4 ones
4. Geometry	Paths and curves, planes, polygons, pyramids, cylinders, cones, spheres Line separates a plane into two half planes
5. Properties & techniques of operations	
a. Natural numbers	
b. Whole numbers	Addition and multiplication binary operations stressing commutative, associative, closure properties of addition, multiplication; inverse relationship of addition and subtraction $325 + 279 = N, 604 - N = 279$ Inverse operation of multiplication to division
c. Fractions	Addition, subtraction with like denominators and
d. Integers	simple fractions whose forms can be made to have like denominators

6. Equalities & inequalities	
a. Sentences	Story problems
b. Symbols	Story problems
Other patterns	Story problems, 2 more than one third of a number is 10. find the number. $\frac{1}{3}$ N + 2 = 10 2 □ 3 = 7 3 □ 5 = 11 Discover the pattern 1 □ 2 = 4 (□ is a symbol of operation)
7. Measurement	Dry measure Perimeter, measure sum of segments Concept of area of rectangle through arrays
8. Statistics	
a. Graphs	Circle graphs, pictographs
Pictographs	One dimension line graphs continued
Circles	Venn diagrams to show intersection and unions
Lines, bar, coordinates	Line graphs to show intersection and unions of sets of numbers such as {1, 2, 3}, {2, 3, 5}

STRANDS	FIFTH GRADE
1. Concepts of Sets	Continued sets of numbers, sets of points, sets of lines
2. Number and Numeral	Prime numbers, composite numbers, equivalent fractional numbers, decimal fractions, ratios–relation to fractional numbers, division, decimals and per cents
3. Enumeration systems	
a. Place value	Enumeration systems in base 2 and base 7
b. Expanded notation	
4. Geometry	Review of forms in Grade 4. Plane separates space into two half spaces
5. Properties & techniques of operations	
a. Natural numbers	
b. Whole numbers	Products and factors, checks of divisibility by 2, 3, 4, 5, 7, 9, 10, 11; greatest common factor, least common multiple

c. Fractions	Addition, subtraction with and without common denominators
d. Integers	Multiplication, whole number times a fraction, a fraction times a fraction
6. Equalities & inequalities	
a. Sentences	Story problems
b. Symbols Other patterns	Equalities, inequalities and patterns dealing with operations as listed
7. Measurement	Concept of area, square units, 1 square inch, 1 square foot Area of the square and rectangle Angle measure, unit 1 degree
8. Statistics	
a. Graphs Pictographs Circles Lines Bar Coordinates	Pictograph and bar graph, scale reading and ratio in two dimension scales line graph or bar graph–

STRANDS	SIXTH GRADE
1. Concepts of Sets	Continued and intersections as multiplication
2. Number and Numeral	Equivalent numbers written in exponential form $4 = (2 \times 2) = 2^2$ Numerals for all integers
3. Enumeration systems	
a. Place value	Other bases to continue the work of place value
b. Expanded notation	Place value, expanded notation in exponential form such as $42{,}506 = 4(10)^4 + 2(10)^3 + 5(10)^2 + (10)^1 + 6(10)^0$

4. Geometry	Closed figures separate sets of points in a plane into interior and exterior, sets and circles Ideas of congruence and side-angle relationships of triangles Ideas of similar triangles Separation of a plane into quadrants Coordinates as an ordered pair
5. Properties & techniques of operations	
a. Natural numbers	
b. Whole numbers	
c. Fractions	
d. Integers	
6. Equalities & inequalities	
a. Sentences b. Symbols Other patterns	Sentences dealing with all operations on equalities and inequalities and more advanced patterns
7. Measurement	Volume; cubic unit – geometric solids Relationships, dry and liquid measure to cubic units 1 gallon = _____ cubic inches
8. Statistics	
a. Graphs Pictographs Circles Lines Bar Coordinates	Organizing and describing data Rate and statistical graphs Scale drawing, scale reading Plotting points on rectangle coordinate plane

CRITERIA FOR EVALUATING BASIC TEXTBOOKS—GRADES 1-8

	Weighting
I. SCOPE OF CONTENT	350

A. Problem Solving and Application

The series of texts shall provide for:

1. Problems which are presented in relationship to processes
2. Problems which are distributed so as to develop and maintain skills in different types of problem solving
3. Problems which are accurate in content and realistic for the age group for which the book is intended
4. Problems which are arranged in gradually increasing order of difficulty
5. Problems which provide adequate content dealing with several types of arithmetic concepts which develop and maintain desired understandings and skills

B. Grade Placement and Sequence

The series of texts shall provide for:

1. Content for each grade which is adapted to the needs, interests, and abilities of pupils
2. The introduction of new processes and skills at equitably distributed intervals throughout all grades
3. Gradation of materials which reflect the current realization that elementary pupils are capable of learning more mathematical content than has been previously introduced
4. A balance among the meaning of the number system, computation, and application and a graduated approach to the structural properties of arithmetic

C. Nature of Number Systems

The series of texts shall provide for:

1. Matching or one-to-one correspondence as the basis for the pupil's instruction in learning to count

2. The definition of operations and their relationships
   a. Subtraction as the inverse of addition
   b. Division as the inverse of multiplication
3. The properties of the operations such as: commutativity, associativity, distributivity, closure, and identity presented in terminology appropriate for elementary pupils
4. The correct use of the terms "number" and "numeral" and symbols appropriate to the ability level of elementary pupils
5. The number line as a visualization for mathematical abstractions
6. Sufficient development of the multiplicative structure of numbers, including the study of factoring and prime numbers
7. Rate, ratio, and per cent presented as special interpretations of rational and real numbers

D. Systems of Numeration and Notation

The series of texts shall provide for:

1. A thorough development of place-value and the decimal system, including the meaning of exponents
2. A consideration of systems of numeration with bases other than ten
3. A study of some historical bases, including Roman numerals, which led to the discovery and utilization of the decimal system

E. Geometry

The series of texts shall provide for:

1. An introduction to the simpler plane configurations and their use in life situations
2. An understanding of the simpler relationships among geometric configurations
3. The construction of simple geometric drawings and models
4. Opportunities to recognize common plane and solid figures and their distinguishing characteristics

F. Measurement

The series of texts shall provide for:

1. Development of the concept of measurement as an association of a number with an object through comparison with an arbitrarily chosen unit
2. A study of common standard units, including those of the metric system
3. Opportunities to convert measures to different units
4. Instruction in estimating in measurement
5. Computation with approximate data including the ideas of precision and accuracy
6. Drawing to scale

G. Mathematical Sentences

The series of texts shall provide for:

1. The use of mathematical sentences in the presentation of number facts
2. The use of mathematical sentences in problem solving
3. The use of mathematical sentences as an aid in stating mathematical principles precisely

H. Graphs and Scale Drawings

The series of texts shall provide for:

1. Graphs and scale drawings presented in such a manner that the pupils understand the mathematical significance of them
2. The use of graphs and scale drawings in practical, life-like situations
3. Graphs and scale drawings which are within the range of interest and understanding of elementary pupils

I. Logic

The series of texts shall provide for an informal treatment, at a level that has meaning for pupils, of such concepts of logical thinking as definition, implication, and quantification

J. **Sets**

The series of texts shall provide for:

1. An informal introduction of the set concept as a means of securing more precise definitions in mathematics
2. An introduction to the terminology and notation of sets appropriate to the grade level

Weighting

II. MANNER OF PRESENTATION 250

The series of texts shall provide for:

A. Pupil involvement and pupil exploration leading to discovery of mathematical principles and procedures

B. The introduction of processes in meaningful situations that will be clear and obvious to elementary pupils:

1. The pupils should be led through a sequence of steps or questions which result in discovery rather than to follow a prescribed series of directions
2. Each step involved in a process should be well explained, providing at all times for the pupil to discover reasons for the step
3. Manipulative and illustrative material should be suggested and/or included to help pupils discover and understand each new process
4. The algorisms of arithmetic should be presented in a mathematically meaningful manner

C. Opportunities for pupils to discover different ways of performing an operation

D. The introduction of arithmetical terms with adequate explanation. The terms should be used consistently throughout the text

E. Opportunities for pupils to experiment and discover many ways to make reasonable estimates of correct answers

F. Opportunities for pupils to develop effective approaches to problem solving

G. Opportunities for pupils to check their own work for accuracy

H. Experience for mental computation

Weighting

III. TEACHER'S EDITION 100

A teacher's edition is essential. It shall include:

A. The pupils' text material and teacher aids

B. Statement of purpose and objectives for each lesson

C. Suggestions for concrete representation when appropriate

D. Adequate information concerning the mathematical background underlying any given lesson

E. Suggestions for presenting each lesson

F. Suggestions for providing for individual differences

G. Suggestions for evaluating pupil progress

H. An index

Weighting

IV. READABILITY 50

Each book of the series of texts shall provide for:

A. A vocabulary which is suited to the grade level for which it is intended

B. A sentence structure that is well adapted for understanding by pupils of the grade level for which it is intended

C. A clear, concise explanation of mathematical terminology

Weighting

V. PROVISION FOR INDIVIDUAL DIFFERENCES 75

The series of texts shall provide:

A. Activities designed to meet the normal range of ability within a class

B. Exercises and problems within a lesson which are graduated toward increasing difficulty

C. Activities which reinforce concepts developed for the slower learner

D. Activities designed to challenge the more capable learner

Weighting

VI. PRACTICE MATERIALS AND MAINTENANCE 50

The series of texts shall provide for:

A. Practice material distributed according to known learning difficulties

B. Sufficient practice for initial mastery

C. Practice materials properly distributed for maintenance and reteaching

D. Sufficient variety of practice materials to stimulate interest

Weighting

VII. EVALUATION AND RETEACHING 75

The series of texts shall provide for:

A. Ample evaluating materials for each aspect of instruction

B. Periodic testing of large units

C. Continued reinforcement of learning and thorough review

D. Diagnostic tests and exercises which point to specific instructional needs and which may refer to content of previous grades

E. Practical self-checking devices for pupils

F. Tests and remedial materials which are designed for practical and economical use by teachers and pupils

Weighting

VIII. PHYSICAL FEATURES 50

The series of texts shall have:

A. **Format**

1. Appropriate spacing and arrangement of pages
2. Illustrations which enhance the learning process and emphasize mathematical concepts
3. Captions to aid learning and to contribute to the teachability of the text
4. Type size and style suitable to grade level
5. Color appropriately used to aid in teaching and learning

B. **Index, Table of Contents, Glossary, References**

ELEMENTARY TEXTBOOKS ADOPTED BY THE STATE BOARD OF EDUCATION

Grades 1-8

Adopted July 1, 1964

MATHEMATICS—Grades 1-8 Series

American Book Company—1963 Edition
Ginn and Company—1963 Edition
Harcourt, Brace, & World—1962 Edition
Holt, Rinehart, & Winston—1963 Edition
Laidlaw Brothers, Inc.—1965, 1963
Science Research Associates, Inc.
Scott, Foresman, & Company—1963 Edition
Silver Burdette Company—1963, 1961
Webster Division McGraw-Hill Book Company—1962 Edition

MATHEMATICS—Grades 7-8 Series

Addison-Wesley Publishing Company—1963, 1961 Edition
Prentice-Hall, Inc.—1964 Edition

SUPPLEMENTARY—Grades 6, 7

Scott, Foresman & Company—1960, 1961
Seeing Through Arithmetic—Special Books A and B

SOURCES OF INFORMATION AND MATERIALS

Ball State Teachers College Experimental Program: Charles Brumfiel, Room 3220, Angell Hall, Department of Mathematics, University of Michigan, Ann Arbor, Michigan. (Inquires about publications of the Program should be addressed to Charles Fleenor, Assistant Professor of Mathematics, Ball State Teachers College, Muncie, Indiana.)

Boston College Mathematics Study: Stanley J. Bezuszka, S.J., Boston College Mathematics Institute, Boston College, Chestnut Hill 67, Massachusetts.

Commission on Mathematics, College Entrance Examination Board; c/o Educational Testing Service, Box 592, Princeton, New Jersey: Report of the Commission on Mathematics, *Program for College Preparatory Mathematics* and *Appendices.*

Committee on the Undergraduate Program in Mathematics, Mathematical Association of America. Dr. B. E. Rhoades, Associate Director, Post Office Box 1000, Pontiac, Michigan. *Recommendations for the Training of Teachers of Mathematics; Mathematics Text Materials for the Undergraduate Preparation of Elementary School Teachers; Course Guides for the Training of Teachers of Elementary School Mathematics.*

Greater Cleveland Mathematics Program: George H. Baird, Educational Research Council of Greater Cleveland, 75 Public Square, Cleveland 13, Ohio.

Minnesota National Laboratory for the Improvement of Secondary School Mathematics: P. C. Rosenbloom, Department of Education, 301 State Office Building, St. Paul, Minnesota.

NASDTEC-AAAS STUDIES, 1515 Massachusetts Avenue, N.W., Washington 5, D. C. *Guidelines for Science and Mathematics in the Preparation Program of Elementary School Teachers.*

National Council of Teachers of Mathematics: 1201 Sixteenth Street, Northwest, Washington 6, D. C. *The Revolution in School Mathematics:* A Challenge for Administrators and Teachers. 50¢. Request publications, price list and membership information.

National Education Association: 1201 Sixteenth Street, N.W., Washington 6, D. C. *Current Curriculum Studies in Academic Subjects:* A Report Prepared for the Project on Instruction.

Rockefeller Report on Education: Doubleday and Company, Inc., Garden City, New York. *The Pursuit of Excellence, Education and the Future of America.*

School Mathematics Study Group: E. G. Begle, School of Education, Stanford University, Stanford, California.

Stanford University Study, Geometry for Primary Grades Project: Newton S. Hawley, Applied Mathematics and Statistics Laboratory, Stanford University, Stanford, California.

Stanford University Study, Sets and Numbers Project: Patrick Suppes, Institute for Mathematics Studies in the Social Sciences, Stanford University, Stanford, California.

Syracuse-Webster Elementary Mathematics Project, Webster College: Robert B. Davis, Webster College, St. Louis 19, Missouri.

University of Illinois Arithmetic Project: David A. Page, University of Illinois Arithmetic Project, 1207 West Stoughton Avenue, Urbana, Illinois.

University of Illinois Committee on School Mathematics: Max Beberman, UICSM Project, 1208 West Springfield, Urbana, Illinois.

University of Maryland Mathematics Project: John R. Mayor, College of Education, Skinner Building, T114, University of Maryland, College Park, Maryland.

GLOSSARY

Algorism or Algorithm: a special pattern of procedure for solving a certain type of problem.

Associative property: that law of a mathematical system which permits the change of grouping in any sum of several terms or in any product of several factors without altering the end result, e.g., $(2 + 3) + 4 = 2 + (3 + 4)$ or $(2 \times 3) \times 4 = 2 \times (3 \times 4)$

Binary Operation: that characteristic which permits an operation to be performed on only two elements of a set at a time.

Cardinal Number: a number used to tell how many members or elements are in a set.

Closure: that condition existing when an operation is performed on any two members of a set and the result is always a member of the set.

Commutative Property: that law of a mathematical system that permits the change of order of a binary operation without altering the end result, e.g., $3 + 4 = 7$ or $4 + 3 = 7$, $3 + 4 = 4 + 3$; $2 \times 4 = 8$ or $4 \times 2 = 8$, $2 \times 4 = 4 \times 2$.

Composite Number: any positive integer which has factors other than itself and 1, e.g., 8, $2 \times 4 = 8$.

Counting Numbers: positive whole numbers used in counting, e.g., 1, 2, 3, 4, 5 $\cdots$

Distributive Property: the distributive property of multiplication with respect to addition asserts that the product of the multiplier and the sum of two or more addends is equal to the sum of the products of the multiplier times the separate addends, eg., $2(3 + 4) = 2 \times 3 + 2 \times 4$.

Empty Set: the set that has no members, whose cardinality is zero.

Factor: any of two or more quantities which form a product when multiplied together.

Identity Element: that element of a set when used in a binary operation with another member of the set gives the result of the other member, e.g., $0 + 3 = 3$, 0 is the identity element; $1 \times 4 = 4$, 1 is the identity element.

Integers: the set of numbers that contains $\cdots -3, -2, -1, 0, 1, 2, 3, \cdots$

Intersection of Sets: a set containing only those elements that are common to the sets under discussion.

Natural Numbers: same as counting numbers.

Non-decimal Numeration System: a way of naming numbers in an orderly sequential manner, using some base other than base 10; the base of a numeration system is the fundamental counting group for number notation in any positional or place-value system.

Number: an abstraction denoting the concept of quantity; an idea that is conveyed by a vocal sound and/or symbol.

Numeral: symbol used to denote a number.

Numeration System: a way of naming numbers in an orderly sequential manner.

Operation: a specific process for combining two or more quantities.

Ordinal Number: a number used to designate the position of a member of a set.

Place Value: term denoting the value assigned to a digit by virtue of its position in relation to the one's place.

Prime Number: any positive integer except the number 1, which has only itself and 1 as a factor, e.g., $7, 1 \times 7 = 7$.

Set: Undefined. May be thought of as a collection, group, or family of objects or ideas which are referred to as elements or members.

Union of Sets: a set containing each element of the sets under consideration at least once but not more than once.

Whole Numbers: non-negative integers. The set of counting numbers and zero.